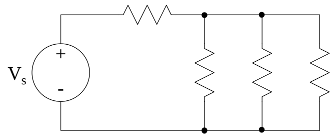
Circuits, Ohm's Law, and Kirchoff's Laws

Circuits

- A circuit is composed of elements (sources, resistors, capacitors, inductors) and conductors (wires).
- Elements are lumped.
- Conductors are perfect.

Schematic



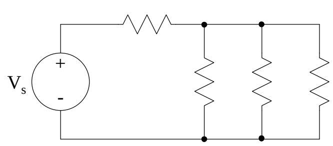
Schematic

- A schematic diagram is an electrical representation of a circuit.
- The location of a circuit element in a schematic may have no relationship to its physical location.
- We can rearrange the schematic and have the same circuit as long as the connections between elements remain the same.

Nodes

To find a node, start at a point in the circuit. From this point, everywhere you can travel by moving along perfect conductors is part of a single node.

Example: Find the Nodes



More Nodes

- Since the node is composed of perfect conductors, the voltage (with respect to an appropriate reference) anywhere in a node is the same.

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Ohm's Law

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Resistors

- A resistor is a circuit element that dissipates electrical energy (usually as heat).
- Real-world devices that are modeled by resistors: incandescent light bulb, heating elements (stoves, heaters, etc.), long wires
- Parasitic resistances: many resistors on circuit diagrams model unwanted resistances in transistors, motors, etc.

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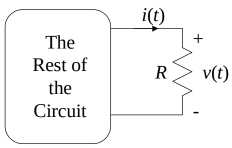
The Mathematical Model



- Resistance measured in Ohms (Ω)
- $v(t) = i(t)\,R$ - or - $V = IR$
- $p(t) = i^2(t)\,R = v^2(t)/R$

Example: the 25W Bulb

- If the voltage across a 25W bulb is 120V, what is its resistance?
- What is the current flowing through the 25W bulb?

Thought Question

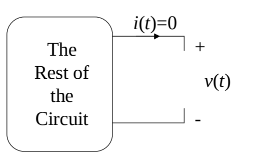
- When I measured the resistance of a 25W bulb, I got a value of about 40Ω. What's wrong here?

Thought Answer

- The resistance of a wire increases as the temperature increases. For tungsten, the temperature coefficient of resistivity is $4.5 \times 10^{-3}/^{o}K$. A light bulb operates at about $5000^{o}F$.

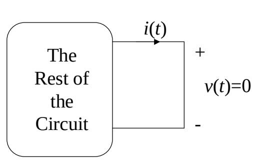
Open Circuit

- What if $R=\infty$?



Short Circuit

- What if $R=0$?



Kirchoff’s Laws

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Summary

Kirchoff’s Current Law (KCL) and Kirchoff’s Voltage Law (KVL) are fundamental properties of circuits that make analysis possible.

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Summary

- KCL
  - sum of all currents entering a node is zero
  - sum of currents entering node is equal to sum of currents leaving node
- KVL
  - sum of voltages around any loop in a circuit is zero

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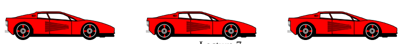
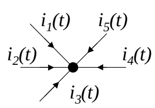
KCL Analogy

- Think of a node as being similar to an intersection on a roadway.
- The number of cars entering the intersection must be equal to the number of cars leaving the intersection or else cars will accumulate in the intersection.



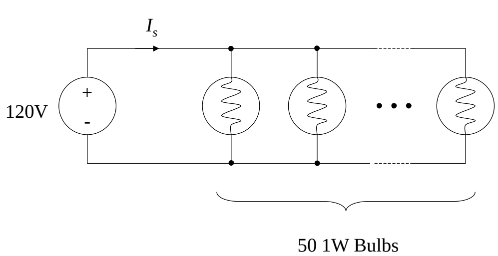
KCL Mathematically



The sum of currents entering the node is zero:

$$\sum_{j=1}^{n} i_j(t) = 0$$

KCL-Christmas Lights



To Solve for I_s:

- Find currents through each light bulb:

  I_B = 1W/120V = 8.3mA
- Apply KCL to the top node:

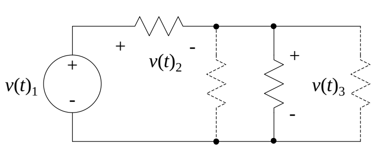
  $I_S - 50I_B = 0$
- Solve for I_S:

  $I_S = 50\ I_B$ = 417mA

KVL Analogy

Applying KVL is analogous to taking a walk while paying attention to increases and decreases in altitude. If you walk in a loop (ie. you end your walk where you started) the net change in altitude is zero. This is true for any path that you take for your walk.

KVL



- The sum of voltages around a loop is zero:

$$\sum_{j=1}^{n} v_j(t) = 0$$

Important Stuff

- A loop is any closed path through a circuit in which no node is encountered more than once.
- A voltage encountered + to - is positive.
- A voltage encountered - to + is negative.

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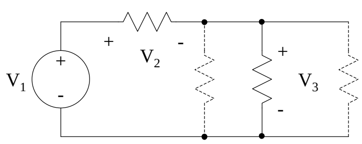
Example-KVL around loop



$-V_1 + V_2 + V_3 = 0$

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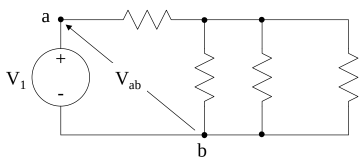
A Different Loop



$-V_1 + V_{ab} = 0$

$V_1 = V_{ab}$

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More Stuff

- Loops need not include circuit elements.
- Arrows represent voltage differences; they point from low to high voltage.

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What would happen if you forgot (or deliberately ignored) the convention that + to - is positive and went around a loop using - to + is positive?

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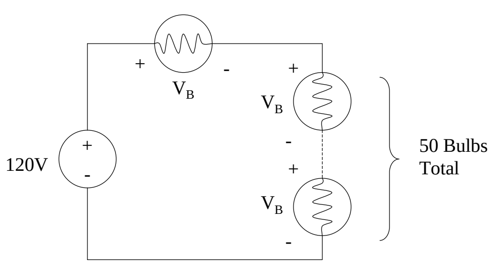
KVL-Christmas Lights



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To Solve for V_B

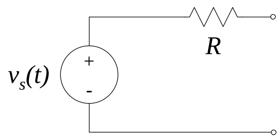
- Apply KVL around the loop:

$$50V_B - 120V = 0$$

- Solve for V_B :

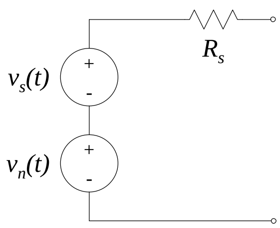
$$V_B = 120V/50 = 2.4V$$

Example: Thermal Noise



R is a resistor in which charged particles vibrate due to random thermal motion.

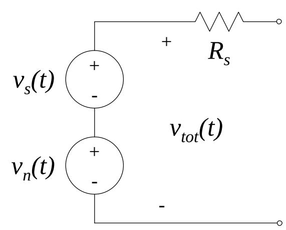
Equivalent Model



More on the Model

- R_S is an ideal resistor (one in which there is no noise).
- $v_n(t)$ represents the thermal noise in the real resistor.

Equivalent Voltage



Solve for $v_{tot}(t)$ using KVL